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| Activity plan | | | | |
| ACTIVITY PLAN | | | | |
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| **Theme** | **Subtopic** | **Activity Title** |
| 3. Creative and Critical Thinking in EcoSTEAM Education | 3.1. Problem-solving in Environmental Contexts | Eco Ellipses: Constructing and applying ellipses to environmental solutions Top of Form |

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| Introduction part (or activity overview) |
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| **Introduction part (or activity overview)** | Students, after learning what an ellipse is and how to independently construct an ellipse using the garden method, will be able to apply their knowledge in designing environmentally . They will solve problems related to the construction and layout of ellipses in real life taking into account environmental factors. |
| **SETTING** | Classroom |

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| Materials Needed |
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| **Materials Needed** | Computer (phone or tablet for interactive applets in GeoGebra), styrofoam, cardboard or drawing paper, video presentation equipment, markers**,** flip charts. |

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| **Learning Outcomes** | * Understanding the definition and properties of the ellipse * Identifying different methods of ellipse construction * Understanding and improvement of using computer programs for drawing mathematical objects * Constructing ellipses with the garden method * Develop problem-solving skills by tackling real-world challenges related to ellipse layout and environmental considerations |
| **Activity Contents** | **ACTIVITY 1 (50 min): Еllipse construction by gardener’s method**  ***Theoretical part 1*** *(10 minutes)*  Through questions, the teacher initiates the students' prior knowledge of an ellipse as a mathematical curve and its application. Students then watch an educational video.  Video: **“What is Ellipse ?”**  <https://www.youtube.com/watch?v=nzwCInIMlU4> (duration 5minutes 49 seconds)  Overview: An educational video designed to provide a basic knowledge and understanding of the ellipse as a mathematical curve and its real-world application  *Material for Teacher:* After the students watch the video, the teacher highlights the definition and properties of the ellipse:   * An ellipse is the set of all points in a plane such that the sum of their distances from two fixed points is a constant. Each fixed point is called a focus (plural: foci) of the ellipse. * Every ellipse has two axes of symmetry. The longer axis is called the major axis, and the shorter axis is called the minor axis. Each endpoint of the major and minor axis is the vertex of the ellipse. The axis are perpendicular at the center. The foci always lie on the major axis and the sum of the distances from the foci to any point on the ellipse (the constant sum) is greater than the distance between the foci.   ***Theoretical part 2*** *(5 min)*  There are several methods of constructing an ellipse. Students use a simple one - the gardening method, that has applications outside the classroom. They perform the following experiment: On a flat surface at a certain distance they stick two nails, tie the ends of a thread to the nails, the length of which is greater than the distance between the nails. If the thread is stretched with a pencil, when the thread is really stretched, the pencil will write a curved closed line called an ellipse.  Video: **“How to draw a perfect ellipse ?”**  <https://www.youtube.com/shorts/nKqfHrYFne8> (duration 10 s)  The French mathematician René Descartes (1596-1650), who is considered the founder of analytical geometry, called the mechanically constructed ellipse a "garden construction".  *Task 1 (20 min)*  The gardening construction of an ellipse can be perfomed in the computer program GeoGebra. The following link will take you to an interactive applet that simulates the garden construction of the ellipse. Move point C on the applet and watch what you get!  **Аn interactive applet 1 in GeoGebra :** <https://www.geogebra.org/m/e5dkn33t>  The students are given instructions about the exercise, then they independently explore the applet, complete the question sheet and, finally, self-assess :  The students can change the position of some of the points and observe the changes and record them in the individual work sheet **(Appendix 1)** . The questions will help students to explore and draw conclusions, as well as to evaluate their own achievements from this exercise.  As seen in the applet, the length of the string is the distance between points A and B, that is, the length b. We can change it by moving the endpoints. Between points A and B, point C is arbitrarily chosen. The points F1, F2 are the nails from the "garden construction". We can change their mutual distance by moving the points, but that distance must be smaller than the length of the thread.  Think and answer the following questions:   1. Where is the center and what is the radius of the two circles of the applet? 2. How were points F and G obtained?   By moving point C, which is between points A and B, points F and G will describe an ellipse with their traces. If we want to get a permanent trace, we will call the GMT tool (locus), then we will click on one intersection point, then on point C and on the other intersection point, then on point C.  Explore the applet and answer the following questions:   1. What will happen if the distance between points A and B changes? 2. What if the distance between points A and B is less than the distance between points F1 and F2? 3. What if the distance between the foci is 0? (If F1 and F2 match)   *Task 2 (15 min)*  Students are shown how an ellipse can be constructed with a tool in GeoGebra if we know its foci and one of its points. In doing so, the equation of the ellipse will be written in the algebraic window. In GeoGebra, tangents to an ellipse can be drawn at a given point on the ellipse or drawn from a point that does not belong to the ellipse, and the equations of the tangents will be displayed in the algebra window. The mutual position of a line and an ellipse can be determined and the coordinates of the intersection points can be determined. (The teacher shows how the drawing tools in the program work, and the students try on their devices). After students learn how to construct ellipses and their tangents with a tool, they can see another interesting way of constructing an ellipse using its tangents, shown in the following applet:  **Аn interactive applet 2 in GeoGebra:** <https://www.geogebra.org/m/ufqxt28c>  Move point A on the applet and watch what you get! (you can also turn on animation of point A (on right click))  **ACTIVITY 2 (45 minutes): Elliptical Elegance: Applications of Ellipses in real life**  *Theoretical part 1 (10 min)*  *Material for Teacher :*The teacher asks the students to think about where ellipses have applications in real life. Through a conversation, the students guided by the teacher are reminded that : Аthletic tracks and tennis courts are mostly in the shape of an ellipse; Elliptical rooms and halls are often included in architecture to create an interesting and functional use of space; Elliptical shapes can be used in the planning of ecological spaces such as parks and gardens, where the shape of the ellipse can be used to maximize space and enhance environmental design; Antennas on satellite systems are often designed with an ellipse shape to allow the signal to be focused in a specific region; Focal lenses in cameras and telescopes are often elliptical in shape for better focusing and detailed view of objects; Ellipses are especially important in astronomy; The orbits of the planets around the Sun are elliptical.  The following video explains the importance of ellipses for Kepler's first law :  Video: **“Kepler’s first law of motion – elliptical orbits (Astrnomy) ”**  <https://www.youtube.com/watch?v=qDHnWptz5Jo> (duration 3minutes 18 seconds)  Overview: An educational video to learn and understand that with his first law of planetary motion, Kepler rejected circular orbits and showed that an ellipse could better explain the observed motions of Mars. Generalized to all planets, it states that the orbit of a planet follows an ellipse with the Sun at one focus.  *Task 1 (30 min)*  Teacher divides the students into 4 groups and assigns each group a specific real-life application of ellipses challenge.   1. The first group works on the problem "Elliptical gears in engineering" 2. The second group works on the problem "Elliptical windows in architecture" 3. The third group works on the problem "Solar panels arranged in elliptical patterns" 4. The fourth group works on the problem "Elliptical pools and garden designs"   Students work in groups to gather information using internet research or literature from their school library and prepare a presentation on their assigned topic. Students can use environmental materials to create posters or digital tools for presentations.  Finally, each group presents its findings to the class.  Other students ask questions and engage in discussions about the various applications.  *Discussion and reflection (5 min)*   * Students review the definition of an ellipse and highlight its real-world application. * Students discuss the challenges they faced during the construction and design process. * The importance of integrating mathematics, environmental science, and art in understanding and designing sustainable solutions is emphasized. |
| **Assessments** | Verbal feedback during class;  Conversation with/among students;  Monitoring of students during group work;  Evaluation of the thoroughness and accuracy of individual work;  Each student self-assesses his contribution to the work;  Aassesments the presentation of the work; |
| **Key Competences** | * Cognitive competence * Creativity competence * Communication competence * Social, emotional and healthy living competences * Digital competence * Practical skills in planning and constructing |
| **Connections with Eco STEAM** | **Eco**- By designing solar panels arranged in elliptical patterns and decorating the ellipses using eco-friendly materials, students will learn valuable skills while promoting environmental awareness.  **S**cience: Students will understand that the orbits of the planets around the Sun are elliptical.  **T**echnology: Students will use various digital programs for ellipse construction and simulations of the changes in the parameters of the ellipses  **E**ngineering: Using the Wireframe method to draw ellipses and researching elliptical gears can help engineers accurately represent these shapes in their plans and models.  **A**rt: Students will design courtyards and gardens with oval shapes  **M**ath: Students will learn to define and construct the mathematical ellipse curve |
| **References** | * Mathematics textbook for high school education in the Republic of Nort Macedonia * Master's thesis: "Application of the GeoGebra computer package in the study of analytical geometry" Author: Aleksandra Arsovska, UKIM Skopje * <https://www.geogebra.org> * <https://courses.lumenlearning.com/waymakercollegealgebra/chapter/equations-of-ellipses/> |
| **Notes** | * Students can construct the ellipses in other computer programs or CAD applications. * Projects that students have to work on in a group can be given as homework for a certain period of time, because it takes more time to think and create a quality design plan. They can make the designs on paper or in a computer drawing program, they can even print them on a 3D printer. |

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| Appendix 1. STUDENT'S individual ACTIVITY SHEET | | | | | | |
| Explore the applet and answer the following questions:  Name and surname: | | | | | | |
| 1.Where is the center and what is the radius of the two circles of the applet? |  | | | | | |
| 2.How were points F and G obtained? |  | | | | | |
| 3.What will happen if the distance between points A and B changes? |  | | | | | |
| 4.What if the distance between points A and B is less than the distance betweenpoints F1 and F2? |  | | | | | |
| 5. What if the distance between the foci is 0? (If F1 and F2 match) |  | | | | | |
| Аfter answering the questions, compare them with the answers given by the teacher and with the help of the questions that follow, evaluate your achievements in this exercise: | | | | | | |
| How well did you understand how the applet works? | | 1 | 2 | 3 | 4 | 5 |
| How much of the applet did you understand what the definition of an ellipse is? | | 1 | 2 | 3 | 4 | 5 |
| How well did you understand how the shape of the ellipse changes as the parameters change? | | 1 | 2 | 3 | 4 | 5 |
| How satisfied are you with your own engagement in masteringnew material? | | 1 | 2 | 3 | 4 | 5 |

**Assessment Table for individual work:**

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| Assessment Criteria | Points | Comments |
| Understanding the definition of the ellipse | \_\_/5 |  |
| Understending the propertiesof the ellipse | \_\_/5 |  |
| Constructing an ellipse with the garden method | \_\_/10 |  |
| Understanding the applications of the ellipse in real life | \_\_/5 |  |
| Digital skills at work in the interactive applet | \_\_/5 |  |
| Quality of the aesthetic and artistic constructions | \_\_/5 |  |

**Assessment Table for group work:**

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| Assessment Criteria | Points | Comments |
| Internet research skills | \_\_/5 |  |
| Dexterity and creativity in the application of ellipses in the project | \_\_/5 |  |
| Ellipse construction skills | \_\_/5 |  |
| Ecological Interpretations in the project | \_\_/5 |  |
| Teamwork and Collaboration | \_\_/5 |  |
| Skills of presenting the work | \_\_/5 |  |